

1-3 Open Sentences (Pages 16–20)

Mathematical statements with one or more variables are called **open sentences**. An open sentence is neither true nor false until the variable has been replaced by a value. Finding a replacement for the variable that results in a true sentence is called **solving the open sentence**. This replacement is called a **solution** of the open sentence. A sentence that contains an equals sign (=) is called an **equation**. A sentence that has the symbols <, >, ≤, or ≥ is called an **inequality**. A **set** of numbers from which replacements for a variable may be chosen is called a **replacement set**. Each object or number in a set is called an **element**, or member. The **solution set** of an open sentence is the set of all replacements for the variable that make the sentence true.

Examples

- a. Is the equation $3a + 12 = 25$ true if $a = 4$?

$$3a + 12 = 25$$

$$3(4) + 12 = 25 \quad \text{Replace } a \text{ with } 4.$$

$$12 + 12 = 25 \quad \text{Multiply 3 by 4.}$$

$$24 \neq 25 \quad \text{Since 24 is not equal to 25, the equation is not true for the replacement value of 4.}$$

- b. Find the solution set for the inequality $7b + 2 \geq 37$ if the replacement set is {3, 4, 5, 6}.

Replace b with	$7b + 2 \geq 37$	True or False?
3	$7(3) + 2 \geq 37 \rightarrow 23 \geq 37$	false
4	$7(4) + 2 \geq 37 \rightarrow 30 \geq 37$	false
5	$7(5) + 2 \geq 37 \rightarrow 37 \geq 37$	true
6	$7(6) + 2 \geq 37 \rightarrow 44 \geq 37$	true

Therefore, the solution set is {5, 6}.

Try These Together

1. Is the equation $x + \frac{1}{3} = \frac{1}{4} + \frac{3}{4}$ true if $x = \frac{1}{2}$?

2. Find the solution set for $3g - 2 < 16$ if the replacement set is {2, 4, 6, 8}.

Practice

State whether each equation is true or false for the value of the variable given.

3. $a + \frac{1}{8} = \frac{6}{8} + \frac{1}{4}, a = \frac{7}{8}$

4. $4x^2 + 2(5) = 40, x = 4$

5. $2x^2 + 3(2) = 56, x = 5$

6. $\frac{1}{g^2 + 1} \leq \frac{1}{5}, g = 2$

Find the solution set for each inequality. The replacement set is $y = \{5, 10, 15, 20\}$.

7. $y - 3 \leq 13$

8. $y + 2 > 10$

9. $3y - 12 \geq 15$

10. **Standardized Test Practice** Which of the following is the solution set for the inequality $3x^2 + 4(2) \leq 56$ if the replacement set is {2, 3, 4, 5, 6, 7}?

A {5, 6, 7}

B {2, 3, 4}

C {4, 5, 6}

D {3, 4, 5}

Answers: 1. false 2. {2, 4} 3. true 4. false 5. true 6. true 7. {5, 10, 15} 8. {10, 15, 20} 9. {10, 15, 20} 10. B