11-1 Simplifying Radical Expressions

(Pages 586–592)

Product Property of Square Roots	For any numbers a and b, where $a \geq 0$ and $b \geq 0$, $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$.
Quotient Property of Square Roots	For any numbers a and b where $a \geq 0$ and $b > 0$, $\sqrt{\dfrac{a}{b}} = \dfrac{\sqrt{a}}{\sqrt{b}}$.
Rationalizing the Denominator	Use the following steps (called **rationalizing the denominator**) to remove a radical from the denominator of a fraction. $\dfrac{\sqrt{a}}{\sqrt{b}} = \dfrac{\sqrt{a}}{\sqrt{b}} \cdot \dfrac{\sqrt{b}}{\sqrt{b}}$ or $\dfrac{\sqrt{ab}}{b}$, where $a \geq 0$ and $b > 0$
Conjugates	The binomials $a\sqrt{b} + c\sqrt{d}$ and $a\sqrt{b} - c\sqrt{d}$ are called **conjugates** of each other. You can use the fact that $(a\sqrt{b} + c\sqrt{d})(a\sqrt{b} - c\sqrt{d}) = a^2b - c^2d$ to produce a product without radicals.
Radicals and Absolute Values	When finding the principal square root of an expression containing variables, be sure that the result is not negative. Use absolute value to ensure nonnegative results where necessary. $\sqrt{x^2} = \lvert x \rvert \quad \sqrt{x^3} = x\sqrt{x} \quad \sqrt{x^4} = x^2 \quad \sqrt{x^5} = x^2\sqrt{x} \quad \sqrt{x^6} = \lvert x^3 \rvert$
Simplest Radical Form	A radical expression is in simplest form when the following three conditions have been met. 1. No **radicands** (the expressions under the radical signs) have perfect square factors other than 1. 2. No radicands contain fractions. 3. No radicals appear in the denominator of a fraction.

Try These Together

Simplify. Leave in radical form and use absolute value symbols when necessary.

1. $\sqrt{84}$ **2.** $\sqrt{90}$ **3.** $\sqrt{125x^2}$

HINT: Find the prime factorization of the number under the radical sign, then simplify the perfect squares. For example, $\sqrt{12} = \sqrt{2 \cdot 2 \cdot 3} = \sqrt{4} \cdot \sqrt{3} = 2\sqrt{3}$.

Practice

Simplify. Leave in radical form and use absolute value symbols when necessary.

4. $\sqrt{156}$ **5.** $\sqrt{270}$ **6.** $\sqrt{800}$ **7.** $\sqrt{44b^5}$

8. $\sqrt{\dfrac{b^2}{49}}$ **9.** $\sqrt{\dfrac{36}{z^3}}$ **10.** $\sqrt{\dfrac{16}{x^2}}$ **11.** $2x\sqrt{8} \cdot 8x\sqrt{7}$

12. **Standardized Test Practice** Simplify $(y + \sqrt{5})(y - \sqrt{5})$.

     A $y^2 - 2\sqrt{5} - 5$ **B** $y^2 - 2\sqrt{5}$ **C** $y^2 - \sqrt{10}$ **D** $y^2 - 5$

Answers: 1. $2\sqrt{21}$ 2. $3\sqrt{10}$ 3. $5\lvert x \rvert\sqrt{5}$ 4. $2\sqrt{39}$ 5. $3\sqrt{30}$ 6. $20\sqrt{2}$ 7. $2b^2\sqrt{11b}$ 8. $\dfrac{\lvert b \rvert}{7}$ 9. $\dfrac{6\sqrt{z}}{z^2}$ 10. $\dfrac{4}{\lvert x \rvert}$ 11. $32x^2\sqrt{14}$ 12. D

Chapter 11