

11-5 The Distance Formula (Pages 611–615)

You can use the Distance Formula, which is based on the Pythagorean Theorem, to find the distance between any two points on the coordinate plane.

The Distance Formula	The distance between any two points with coordinates (x_1, y_1) and (x_2, y_2) is given by the following formula: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
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Examples

- a. Find the distance between (2, 3) and (6, 8).**

Let $x_1 = 2$, $y_1 = 3$, $x_2 = 6$, and $y_2 = 8$.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d = \sqrt{(6 - 2)^2 + (8 - 3)^2}$$

$$d = \sqrt{4^2 + 5^2}$$

$$d = \sqrt{16 + 25}$$

$$d = \sqrt{41} \text{ or about } 6.4 \text{ units.}$$

- b. Find the value of a if $(a, 3)$ and $(2, -1)$ are 5 units apart.**

Let $x_1 = a$, $y_1 = 3$, $x_2 = 2$, $y_2 = -1$, and $d = 5$.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$5 = \sqrt{(2 - a)^2 + (-1 - 3)^2}$$

$$5 = \sqrt{(-a + 2)^2 + (-4)^2}$$

$$5 = \sqrt{a^2 - 4a + 4 + 16}$$

$$5 = \sqrt{a^2 - 4a + 20}$$

$$5^2 = (\sqrt{a^2 - 4a + 20})^2$$

$$25 = a^2 - 4a + 20$$

$$0 = a^2 - 4a - 5$$

$$0 = (a + 1)(a - 5) \quad \text{Factor.}$$

$$a = -1 \text{ or } a = 5 \quad \text{Zero product property}$$

Practice

Find the distance between each pair of points whose coordinates are given. Express answers in simplest radical form and as decimal approximations rounded to the nearest hundredth if necessary.

- $(4, 6), (1, 5)$
- $(15, 4), (10, 10)$
- $(-7, -2), (11, 3)$
- $(6, 13), (2, 15)$
- $(25, 11), (18, 6)$
- $(12, 3\sqrt{5}), (6, 2\sqrt{5})$

Find the value of a if the points with the given coordinates are the indicated distance apart.

- $(1, 3), (a, -9); d = 13$
- $(-5, a), (3, -7); d = 10$
- $(-9, 3), (-2, a); d = \sqrt{74}$

- 10. Geometry** Find the perimeter of square $QRST$ if two of the vertices are $Q(5, 9)$ and $R(-4, -3)$.

- 11. Standardized Test Practice** Find the distance between the points whose coordinates are $(2\sqrt{7}, 4\sqrt{5})$ and $(\sqrt{7}, 2\sqrt{20})$.

A $\sqrt{5}$

B $\sqrt{7}$

C $\sqrt{32}$

D $\sqrt{70}$

Answers: 1. $\sqrt{10}$ or 3.16 2. $\sqrt{61}$ or 7.81 3. $\sqrt{349}$ or 18.68 4. $2\sqrt{5}$ or 4.47 5. $\sqrt{74}$ or 8.60 6. $\sqrt{41}$ or 6.40 7. -4 or 6