

12-7

Rational Expressions with Unlike Denominators

(Pages 678–683)

The **least common multiple** (LCM) of two or more positive whole numbers is the least positive number that is a common multiple of all the numbers. To add or subtract rational expressions with unlike denominators, first rename the fractions so the denominators are alike, using the least common denominator of the fractions. You may need to factor one or both of the denominators first. The **least common denominator** (LCD) is the LCM of the denominators.

Examples

a. Find $\frac{5}{2y} + \frac{4}{3y^2}$.

List the prime factors of $2y$ and $3y^2$ to find the LCD.

$$2y = 2 \cdot y \quad 3y^2 = 3 \cdot y \cdot y$$

Use each prime factor the greatest number of times it appears in each of the factorizations.

LCD: $2 \cdot 3 \cdot y \cdot y$ or $6y^2$

Change each rational expression into an equivalent expression with the LCD.

$$\begin{aligned} \frac{5}{2y} + \frac{4}{3y^2} &= \frac{5(3y)}{2y(3y)} + \frac{4(2)}{3y^2(2)} \\ &= \frac{15y}{6y^2} + \frac{8}{6y^2} \text{ or } \frac{15y + 8}{6y^2} \end{aligned}$$

b. Find $\frac{x}{x-1} - \frac{5}{x-2}$.

LCD: $(x-1)(x-2)$

$$\begin{aligned} \frac{x}{x-1} - \frac{5}{x-2} &= \frac{x(x-2)}{(x-1)(x-2)} - \frac{5(x-1)}{(x-1)(x-2)} \\ &= \frac{x^2 - 2x}{(x-1)(x-2)} - \frac{5x - 5}{(x-1)(x-2)} \\ &= \frac{x^2 - 2x - (5x - 5)}{(x-1)(x-2)} \\ &= \frac{x^2 - 7x + 5}{(x-1)(x-2)} \end{aligned}$$

Practice

Find each sum or difference. Express in simplest form.

1. $\frac{1}{2x} - \frac{2}{10x}$
2. $\frac{1}{7x} + \frac{2}{x}$
3. $\frac{10}{xy^2} + \frac{5}{y^2}$
4. $\frac{9}{a^3} - \frac{7}{a}$
5. $\frac{2}{3x+6} + \frac{5}{x+2}$
6. $\frac{7}{2x-8} - \frac{2}{x-4}$
7. $\frac{2x}{x+1} + \frac{x}{4x+4}$
8. $\frac{5}{x+6} + \frac{3}{x+3}$
9. $\frac{4}{x+3} - \frac{5x}{x-3}$
10. $\frac{7x}{x^2-16} + \frac{2}{x+4}$
11. $\frac{x}{x-10} - \frac{3}{x^2-100}$
12. $\frac{4x}{x-1} + \frac{-x}{x^2+5x-6}$
13. **Standardized Test Practice** Find $\frac{3}{x^2+x-20} + \frac{2}{x+5}$.
 - A $\frac{5}{x-4}$
 - B $\frac{5}{x^2+x-20}$
 - C $\frac{2x-5}{x-4}$
 - D $\frac{2x-5}{x^2+x-20}$

Answers: 1. $\frac{10x}{3}$ 2. $\frac{7x}{15}$ 3. $\frac{5x+10}{9}$ 4. $\frac{xy^2}{7x^2+9}$ 5. $\frac{3x+6}{17}$ 6. $\frac{2x-8}{3}$ 7. $\frac{4x+4}{9x}$ 8. $\frac{8x+33}{8x+33}$ 9. $\frac{-5x^2-11x-12}{6-9}$ 10. $\frac{9x-16}{8}$ 11. $\frac{x^2+10x-3}{4x^2+23x}$ 12. $\frac{x^2+5x-6}{x^2+5x-6}$ 13. D